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On hydraulics of capillary tubes

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ABSTRACT

The article considers the laws of motion of water in the capillary tubes, taken as a model for flowing well, on the analogical net count device. For capillary tube the lower limit value of flow rate is empirically determined above which the total hydraulic resistance of the capillary is practically constant. The specificity of the phenomenon is that the regime of motion, by a Reynolds number, for a given flow rate still remains laminar. This circumstance can perplex the specialists, so the author invites them to the scientific debate on the subject of study. Obviously, to identify the resulting puzzle it is necessary to conduct a series of experiments using capillaries of different lengths and diameters and with different values of overpressure. The article states that in tubes with very small diameter the preliminary magnitude of capillary rise of water in the presence of flow plays no role and can be neglected.

Introduction

We present the research results on steady movement under different pressures in a small diameter glass tubes (capillaries) which show that in capillaries after a certain values of the flow rate the hydraulic resistance value, with some accuracy, can be accepted as constant. The specificity of the phenomenon is that in case of the noted value the regime of motion by Reynolds still remains laminar. This fact can perplex professionals, so the author invites them to scientific debate on this subject.

Objectives and methods

On the analogical net count device we used glass capillary tubes as a model of artesian flowing well which allow us to create non-linear hydraulic resistances.

To ensure the problem solution accuracy on the model, as well as to avoid unnecessary complications in the simulation process, it is necessary, that the resistance value remains practically unchanged throughout the period of the solution and independent from the change of flow rates through the capillaries. In other words, in the capillaries there should be provided the hydraulic resistance zone which is mostly present in the flowing wells under natural conditions [1,8].

The proposed requirement means that each certain capillary needs special laboratory studies to determine the minimum value of the flow rate after which, in case of the least amount, a significant resistance change will take place in the capillary [7,9].

In the mathematical modeling the definition of the flow rate minimum value is also important in the sense of that it, in some way, conditions the final selection of the values of pressure and large-scale linear receptivity coefficients of hydraulic modeling [1,8].

Usually, to seek resolution of the simulation, the values of these coefficients are selected according to the discretion of

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the users, according to the technical characteristics of the count device. But under conditions of flowing well the simulation values of these coefficients should be chosen so that the non-linear resistance remains constant along with the steadiness the linear hydraulic resistance values throughout the solution.

The laboratory studies of the capillary used in the future model of the well were carried out on a separate test device the schematic image of which is given in Fig. 1 [4,7].

Results and analysis

Let us consider the water outflow through the capillary under the overpressure. In this case, we have a steady-in-time movement, so we can use Bernoulli's assurance [2,3,5,6].

Let us write it for sections A and B for the comparability plane B. So, we will have:

$$Z_A + \frac{P_A}{\gamma} + \frac{\alpha V_A^2}{2g} = Z_B + \frac{P_B}{\gamma} + \frac{\alpha V_B^2}{2g} + \sum h_L, \tag{1}$$

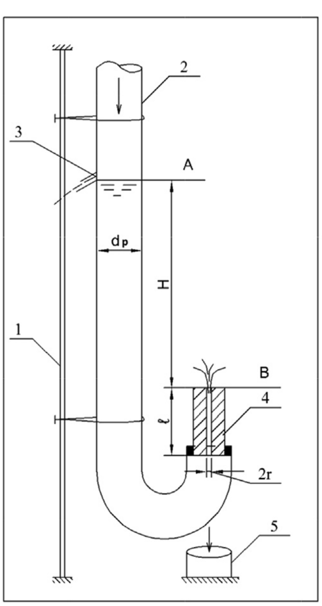
where Z_A- and Z_B are the geometric heights of centers of relevant sections centers from the comparable plane, P_A- and P_B- are absolute pressures in these sections, V_A-the speed in the flexible pipe, V_B- speed in the capillary, Σh_L – sum of the pressure losses along the flow way, it is:

$$\sum h_L = h_{L_1} + h_{L_2}, \tag{2}$$

where h_{L1}-pressure losses in the flexible pipe, h_{L1}-the same in the capillary tube.

From Fig. 1 it easy to notice that $Z_A = H$, $Z_B = 0$ and $P_A = P_{atm}$.

At the initial state, when there is no water flow in the capillary, taking into consideration the capillary water increasing amount under the influence of surface tension forces, for P_B we can write:



1 - Real strut

2 - Flexible rubber tube

3 - Mobile spillway

4 - Capillary

5 - Water volume meter

Fig. 1 – Test scheme.

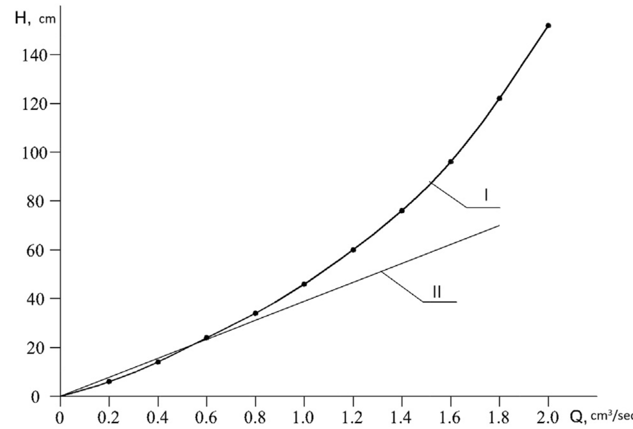


Fig. 2 – Graph of the H = f (Q) function.

$$PB = P_{atm} - \gamma h_c, \tag{3}$$

where h_c is the value of water increasing in the capillary tube and which is suggested to determine by the following relation [3,6]:

$$h_c = \frac{15}{r}, \tag{4}$$

where r is the radius of the capillary.

However, when there is a water stream in the capillary, then at the part of its contact with the air the curved surface and generating surface tension forces are disappearing and therefore we can accept, that $P_B = P_{atm}$.

On the test device the flexible tube diameter (d_t) is ten times bigger than the capillary diameter, hence the speed of the water flow here is relatively slow than in the capillary ($V_A << V_B$), further we designate $V_B = V$. This fact allows with great accuracy to accept $h_{c_1} = 0$.

Based on the scheme in Fig. 1 for h_{c_2} we can write [2,5].

$$h_{c_2} = \left(\xi_{ent} + \lambda \frac{\ell}{2r}\right) \frac{V^2}{2g}, \tag{5}$$

where ℓ is the length of capillary, ξ - the hydraulic resistance coefficient of entrance. Taking into consideration that the flexible tube diameter is $d_t >> 2r$, then with enough accuracy we can admit that $\xi_{ent} = 0{,}5$, λ – the coefficient of hydraulic friction resistance.

$$H = \left(1,5 + \lambda \frac{\ell}{2r}\right) \frac{V^2}{2g}: \tag{6}$$

Taking into consideration the given above assumptions, terms and conditions, from the equation (1) we get:

$$H = \left(1,5 + \lambda \frac{\ell}{2r}\right) \frac{V^2}{2g} \tag{7}$$

Replacing the flow speed by its flow rate, equation (6) will looks like:

$$H = \eta Q^2 \tag{8}$$

where Q is effluent flow rate through the capillary tube, η-the

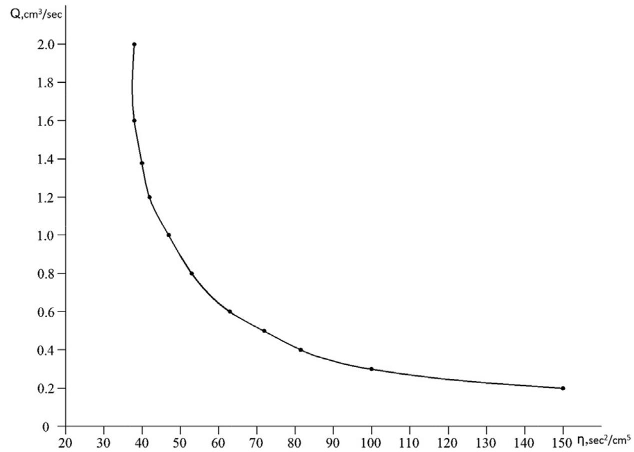


Fig. 3 — η = f (Q).

total hydraulic resistance of the capillary and expressed by the following appointment:

$$\eta = \frac{1,5 + \lambda\frac{\ell}{2r}}{2g\pi^2 r^4} \tag{9}$$

Some laboratory hydraulic studies were carried out on the device suggested in Fig. 1 for the capillaries № 32 (with the following dimensions: length $\ell = 12$ cm and radius r = 0,53 mm) and № 16 ($\ell = 40$ cm and r = 1,0 mm) [4,7]. We measured the effluent flow rate through the capillary tube in compliance with different values of H overpressure. On the device the distilled water as a repulsive fluid is used. By the mentioned measurement data for capillary № 32 we built the graphic curve for H = f (Q) function, which is suggested in Fig. 2 (Line I).

It is obvious that the curve brought in Fig. 2 is a parabola, which is passing through the coordinate initial point, and its nearest section to the top part (in case of the suggested example $0 < Q \le 0,6$ cm^3/s) can be considered as a direct line. This flow rate value is defined as a laminar marginal $Q_{LB} = 0,6$ cm^3/s. Individual observations have shown that this line can be described by *Poiseuille's* formula that the author got as a result of studying venous blood flow movement in the case of laminar regime [3,6]. According to our appointments this formula will look like:

$$H = \frac{8\nu\ell}{\pi g r^4}Q \tag{10}$$

where ν is the fluid's cinematic coefficient of viscosity.

$$H = \frac{8\cdot 0,01\cdot 12}{3,14\cdot 981\cdot 0,0534}Q = 39,5Q\,: \tag{11}$$

Moreover, the equation (9) can be derived easily from the Darcy—Veysbah's formula for calculation of pressure losses, if there will be placed the value $\lambda = \frac{64}{\mathrm{Re}}$, where Re is Reynolds's number. For the observed capillary (water temperature was 23 °C), according to formula (9) for the direct line we will receive the following equation:

$$H = \frac{8\cdot 0,01\cdot 12}{3,14\cdot 981\cdot 0,0534}Q = 39,5Q$$

In Fig. 2 this direct line is presented by Line II. Note, that the appropriate Reynolds number for the flow rate Q will be 720, in the cases of large values there will be deviation from the *Poiseuille's* formula.

Now we will consider the equation (7). With the help of it, by using the measured data of H and Q the values we calculated the value of total hydraulic resistance (η) based on which the dependence curve of $\eta = f(Q)$ was built suggested in Fig. 3. As observed in this curve, after a certain value of flow rate we can accept that the values of η do not change.

In case of the observed capillary № 32 it makes up $Q \ge 1,5$ cm^3/s. We can name it as a turbulent - boundary yield (flow rate), $Q_{TB} = 1,5$ cm^3/s. According to this yield it is derived that η = 38 s^2/cm^5, and the Reynolds number is Re = 1800. In case of capillary № 16 we respectively get that $Q_{TB} = 3,0$ cm^3/s, η = 3,5 s^2/cm^5, Re = 1910.

Thus, for the test capillary № 32 the hydraulic total resistance, which can be practically accepted as constant, makes up 38 s^2/cm^5, and to ensure this during the simulation process, the through-passing yield should not be less than $Q_{TB} = 1,5$ cm^3/s.

As shown in the relation (8) during the flow the change of the value of η only depends on the value of λ and if η remains constant, then it means that the Q-dependent λ no longer changes. This circumstance allows us to insist that in case of flow rate is $Q \ge 1,5$ cm^3/s t, there is a quadratic hydraulic resistance zone in the capillary. However, based on the classic hydraulics it is known that in the pipes there is a quadratic resistance zone generated when there is an intense turbulent movement and the Reynolds number amounts to the several tens of thousands and more. However, as noted above, such condition in the capillary is observed in the cases of quite small values of the Reynolds number, in such conditions the movement in ordinary pipes is expressed as a strongly laminar.

This circumstance can seriously perplex the professionals. To avoid it and to clear up the created contradiction it is also necessary to consider capillaries with diverse characteristics and to carry out comprehensive and detailed studies of fluid movement in them.

The length of capillary required for stream formation can be determined by the following well-known formula [3,6].

$$L = 0,13R_e\,r \tag{12}$$

On calculating the value of Re in compliance with Q_{TB}, then for certain capillaries we will get $L_{32} = 11$ cm < 12 cm and $L_{16} = 20$ cm < 40 cm.

Let us determine the values of λ for two boundary (critical) yields Q_{LB} and Q_{TB} by using relation (8) and then compare the well-known pipe hydraulics formulas for various modes of movement and the derived values.

From the relation (8) we can write for λ:

$$\lambda = \frac{4g\pi^2 r^5\eta}{\ell} - 3\frac{r}{\ell} \tag{13}$$

In the diagram, suggested in Fig. 3, η = 79 s^2/cm^5 corresponds to the yield Q = 0,6 cm^3/s.

According to this, from the relation (12) we receive that λ = 0,09.

For ordinary pipes in case of laminar movement we will have [2,5]:

$$\lambda = \frac{64}{Re} = \frac{64}{720} = 0,089 \tag{14}$$

From the relation (12), when $Q_{TB} = 1,5\ cm^3/s$, we respectively get that $\lambda = 0,038$. Let us calculate the value of λ by the Shifrinsone's formula used for quadratic resistance zones in the ordinary pipes and, accepting that the value of absolute roughness of the glass pipe $\Delta = 0,006$ mm [3,6], we will have:

$$\lambda = 0,11\left(\frac{\Delta}{2}\right)^{0.25} = 0,03 \tag{15}$$

On doing similar calculations for № 16 we get:

when $Q_{LB} = 2,25\ cm^3/s$, corresponding to formula (12) $\lambda = 0,031$, and by (13) $\lambda = 0,044$,
when $Q_{LB} = 3,0\ cm^3/s$, corresponding to formula (12) $\lambda = 0,028$, and by (14) $\lambda = 0,026$.

The comparison of values of λ shows that the formula (12) gives a satisfactory result (in some cases very approximate values should be considered contingency).

Conclusion

The studies of water movement in capillary tubes have shown that there is a deviation from the regularities accepted in the pipe hydraulics and to interpret it is necessary to increase the research posts by including capillary with different diameter and length and by changing in the wide range the pressure that affects the capillary.

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